Mathematics and programming - the easy languages to learn

ATTILA EGRI-NAGY
Akita International University
v2018.09.04 DRAFT at 11:37

Abstract

Clear thinking is needed in everything we do professionally. Mathematics is the study of clear thinking. Therefore, you need maths regardless what your major is. Education did a terribly job in getting this message through. Even worse, it created an image of maths that looks the opposite of what it really is. Not surprisingly, most people hate the subject. If you do, it is not your fault, but now the choice is yours. Opting out means that you choose to be a lesser professional in your field. On the other hand, taking maths courses will equip you with better cognitive tools and make your work easier and more successful.

A cultural disaster

Let's be honest! Chances are that you don't like Mathematics. It is also likely that you don't feel bad about this, since nobody likes maths anyway. In this situation, there are bad news and good news. The bad one is that your dislike of maths is hindering your studies and may damage your future career. This is not your fault, the way you experienced maths might have been unpleasant. The good news is that you can fix this problem.

What is wrong with not taking Mathematics courses? Mathematics is about *precise thinking*. No matter what you think about, if you really want to have a solution to a problem, then sooner or later you start scribbling on a piece of paper to build an *abstract model*. You leave out unnecessary details, identify the core concepts of the problem and consider their relationships. People often do not realize that they are thinking mathematically anyway.

For instance, when planning your holiday (that is certainly not maths, right?!), you might have different options for travel. They may have different time requirements, different costs. Your friends' availability is also something to consider. To clarify your preference, you might find it difficult

to write them in a single list of increasing order. Visiting a big city has different attractions than going on hiking trip, you like them both in a different way. On the other hand, you might clearly prefer one city over the other. Some options are easy to compare, others are not (welcome to the theory of partial orders!). Two trips might be in conflict, or they might fit well together in your holiday time (well, that's graph theory for you).

Of course, you can plan your holiday without knowing a lot of maths. But do you really think that studying the essence of everyday problems in a less complicated settings would not help solving the real problems? Would some running exercise not help you in playing soccer??? Yes, it would. And maths is for building your mental stamina and logical thinking. It's not that you will solve logarithmic equations everyday in your work, but you will certainly need to deal with complex issues where reshaping the problem could give a better path to a solution. Speaking of logarithms, it is ironic that exactly where the mathematics becomes really useful is the moment where the subject looses most of its students.

Maths is about making things easy

Mathematics is an example of an *artificial language*. It is designed, or rather evolved, to have no ambiguity. The meaning for each expression is clearly defined. Whenever we get lost, not knowing what a mathematical expression means, we simply just go back to the definitions. It is like when learning a foreign language, not understanding a sentence due to an unknown word, we simply look that up in a dictionary. This simple technique can give anyone a degree in maths.

In contrast, in natural languages, like English or Japanese, the meaning of a sentence depends on who says it, when and in what situation. In other words, it depends on context, history and culture. It requires more background knowledge and cognitive ability to interpret the situation. Therefore, learning a foreign language requires more intellectual effort than studying maths.

Abstraction is a fundamental operation in Mathematics. It is the act of leaving out unnecessary details from the discussion, aims to make thinking easier. All those fancy symbols, mathematical notation is also used to reduce our cognitive load. Not many people realize that maths can be seen as a very efficient shorthand notation. Everything can be said in plain English, but it would be a very long and complicated sentence to write down the meaning of a clever mathematical symbol. Mathematics is created by people and with the sole purpose of making thinking about hard problems easy.

Why Mathematics is the most hated subject then?

In a way Mathematics is its own victim. Due to its precise nature, it is easy to set up and evaluate standardized tests in it. Unfortunately, it is used as a kind of 'weaponized grading' to knock people off from certain career paths. It becomes highly competitive and promote a sharp division of people who get it, and people who don't. This is very sad, as mathematical research is done exactly the opposite way. It is very social and collaborative.

Another failure of maths education is due to good intentions. Take, for instance, the quadratic equation. It has a special status, since it is powerful enough to model many things in science, and it is accessible enough that anyone can deal with it. There exists a general formula for solving all quadratic equations. Here comes the good intention. The formula is easy to use, just need to plug three numbers in and we get the solutions. So, all we have to do is to give kids to fomula and tell them to use it. We saved them a lot of work, and we made them very clever. How nice! Now look at the kids' perspective. One has to remember the formula, which is just a rather weird combination of symbols. Our brains are not good at memorizing these, we better remember stories. The mechanistic application of the formula removes the need for thinking. Thus solving quadratic equations becomes a dull exercise, with no intellectual reward, just the fear of failure.

Alternatively, we could derive the quadratic formula instead of remembering it. It is doable once you the basic algebraic operations. It allows to grasp the real power of algebra: by using letters instead of concrete numbers, we can solve infinitely many problems in one go. Mathematics is empowering, it gives one the "I can do it!" feeling. It is better to think about *Why?* than *How?* questions only. The lack of explanations is harming.

Mathematics is incremental: advanced concepts build on simpler ones. Therefore, an early missed notion can block the understanding of the rest of the material. Unfortunately, it is very difficult to detect these misconceptions in a traditional classroom setup.

Another mistake we make is emphasizing its usefulness. Well, Mathematics is not useful in that direct sense. You will not solve quadratic equations in your work. But focused thinking with abstractions and different representations of the same problem needs to be practised, and maths is a nice playground for that.

Also, our world is changing quickly. Mathematics remains to be valid, but some new parts are introduced. Standard curriculum is lagging behind. Do you know that there is a mathematical theory of knots?

Opportunity in computer programming

There is now an opportunity for turning the tide. We could teach computer programming and maths in a unified way. These subjects are close to each other. They both use an artificial language for modelling and understanding our world. Mathematics is increasingly difficult to teach without programming. Hand calculated examples are too small for modern applications (like cryptography). It is difficult to motivate the precise use of notation. Students think precision is needed only because the cranky old math professor is not happy without it. When faced with the computer, sloppiness in what we write has immediate consequences.

Roughly speaking, mathematics is a programming language. The 'programs' are executed in our heads, without physical computers. What more, sometimes just the notation itself can do the work. For instance, derivation rules in Calculus will implicitly do limit calculation of functions for us.

Unification brings new meaning to usefulness of mathematics. Functions are central in the maths curriculum. It turns out that the mathematical notion of function is indeed useful in software industry, as the so called functional programming languages are gaining popularity.

Why only an opportunity? Superficial differences do exist, and they are on the surface. Many programming languages have syntax and execution models that are rather different from algebra. Educators barely started to think about how to do this properly.

Opportunity in Liberal Arts

The maths curriculum at Liberal Arts colleges is more flexible then universities specialized in science and technology. It does not have to offer some remedial maths courses (unfortunately, that is often the case). It can bypass much of the more technical material, focusing on the bigger picture. By the same token, Liberal Arts education has a better chance of improving the situation by unifying mathematical and computational thinking in its curriculum.

Similarly, mathematical and scientific subjects also require a broader perspective and they need interaction with other subjects. As our civilization is learning it in the hard way now, technological developments detached from sensitivity of humanities can turn into disastrous forces.

What can you do? Make mathematical courses an integral part of your study plan. In those, always demand for illuminating explanations and be critical about bits of maths presented as a mere algorithm to follow.

Further reading

There are many great books on what mathematics is really about. Trying to list all of them would be overwhelming. Most of them are written for people who are already interested in maths and want to see the interesting parts. Unfortunately, this happens rarely. Therefore, as a personal choice, I select books from two authors, who actually changed their careers from research mathematicians to educators. They try to show people that there is art in maths that can be appreciated by everyone. Their books are the 'fire extinguishing' efforts. Eugenia Cheng writes in a conversational style, seemingly talking about recipes (yes, cooking), but one can learn a lot about the real purpose of mathematics [Cheng, 2015] and about the concept of infinity [Cheng, 2017]. Paul Lockhart wrote an eloquent essay about maths as a form of art [Lockhart, 2009], then he proceeded to demonstrate how maths can be described in an illuminating style about measurement [Lockhart, 2012] and arithmetic [Lockhart, 2017].

TL;DR

Due to unfortunate circumstances, our culture has suffered a terrible accident and got its highest art of clear thinking (i.e. Mathematics) amputated. Institutional changes are slow and uncertain, so the only hope for recovery is in students seeing through the situation and not hating the subject for the wrong reasons.

References

Cheng, 2015. Cheng, E. (2015). *How to Bake Pi: Easy recipes for understanding complex maths*. Profile Books.

Cheng, 2017. Cheng, E. (2017). *Beyond Infinity: An expedition to the outer limits of the mathematical universe*. Profile Books.

Lockhart, 2009. Lockhart, P. (2009). *A Mathematician's Lament*. Bellevue Literary Press.

Lockhart, 2012. Lockhart, P. (2012). *Measurement*. Harvard University Press.

Lockhart, 2017. Lockhart, P. (2017). *Arithmetic*. Harvard University Press.